Homework 7: Polynomials *

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Problem 1. Let k be a fixed positive integer. The n-th derivative of $\frac{1}{x^k-1}$ has the form

$$\frac{P_n(x)}{(x^k-1)^{n+1}},$$

where $P_n(x)$ is a polynomial. Find $P_n(1)$.

Problem 2. Find a nonzero polynomial $P(x,y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$, for all $a \in \mathbb{R}$. (Here $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

Problem 3. Determine all polynomials $P(x)$ such that

$$P(x^2+1) = ((P(x))^2 + 1 \text{ and } P(0) = 0.$$

Problem 4. Let

$$f(x) = az^4 + bz^3 + cz^2 + dz + e = a(z-r_1)(z-r_2)(z-r_3)(z-r_4)$$

where $a, b, c, d, e \in \mathbb{Z}$, $a \neq 0$. Show that if $r_1 + r_2 \in \mathbb{Q}$ and if $r_1 + r_2 \neq r_3 + r_4$ then $r_1 r_2 \in \mathbb{Q}$.

*Due on 11/18/2014, in class.