Homework 5 *

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Problem 1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

$$f(x, y) + f(y, z) + f(z, x) = 0, \ \forall \ x, y, z \in \mathbb{R}.$$

Prove that there exists a function $g : \mathbb{R} \to \mathbb{R}$ such that

$$f(x, y) = g(x) - g(y), \forall \ x, y \in \mathbb{R}.$$

Problem 2. Let

$$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx,$$

where $a_1, a_2 \ldots, a_n$ are real numbers and n is a positive integer. Given that $|f(x)| \leq |\sin x|$ for all real x, prove that

$$|a_1 + 2a_2 + \cdots + na_n| \leq 1.$$

(Hint: You may want to consider f'.)

Problem 3. Find all differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that for any positive integer n

$$f'(x) = \frac{f(x + n) - f(x)}{n}.$$

Problem 4. Basketball star Shanille O'Keal's team statistician keeps track of the number, $S(N)$, of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 50% of N, but by the end of the season, $S(N)$ was more than 50% of N. Was there necessarily a moment in between when $S(N)$ was exactly 50% of N?

(Optional: Can you think of some other values that could be substituted for 50% in the problem?)

*Due on 11/4/2014, in class.