Homework 4: Sequences *

Liubomir Chiriac

Problem 1. Consider a sequence $b(n)$ such that $b(1) = 1, b(2n) = b(n)$, and $b(2n+1) = (-1)^n b(n)$. Evaluate

$$\sum_{n=1}^{2015} b(n)b(n+2).$$

Problem 2. Consider a sequence $\{x_n\}$ with $x_0 = x_1 = x_2 = 1$, and for $n \geq 0$:

$$x_{n+3} = \frac{x_{n+1}x_{n+2} + n!}{x_n}.$$

Show that each x_n is an integer. (By convention, $0! = 1$.)

Problem 3. Consider the power series expansion

$$\frac{1}{1 - 2x - x^2} = \sum_{n=0}^{\infty} a_n x^n.$$

Prove that for each integer $n \geq 0$, there is an integer m such that

$$a_n^2 + a_{n+1}^2 = a_m.$$

Problem 4. Compute

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{m^2 n}{3^m(n3^m + m3^n)}.$$

*Due on 10/28/2014, in class.