

Homework 2: Geometry *

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Problem 1. Given that A, B, C are non-collinear points in the plane with integer coordinates such that AB, BC, CA are all integers, what is the smallest possible value of AB ?

Problem 2. Show that the curve $x^3 + 3xy + y^3 = 1$ contains only one set of three distinct points A, B, C , which are the vertices of an equilateral triangle, and find its area.

Problem 3. Given that the parabolas $y = ax^2 + ax + \frac{1}{24}$ and $x = ay^2 + ay + \frac{1}{24}$ are tangent to each other, find all possible values of a .

Problem 4. Let \mathcal{F} be a finite collection of open discs in \mathbb{R}^2 whose union contains a set $E \subseteq \mathbb{R}^2$. Show that there is a pairwise disjoint subcollection D_1, \dots, D_n in \mathcal{F} such that

$$E \subseteq \bigcup_{j=1}^n 3D_j.$$

(Here, if D is the disc of radius r centered at P , then $3D$ is the disc of radius $3r$ centered at P .)

*Due on 10/14/2014, in class.