PROBLEM SET NO. 6 (DUE ON MONDAY, NOVEMBER 17 AT 4:00 PM)

- **Problem 3:** Consider the following functions defined on all of $\mathbb{R}$,

$$f(x) = x^3 - 3x^2 \ , \ g(x) = \frac{x+1}{x^2+1} \ .$$

  (i) Are f, g injective? If not, decompose $\mathbb{R}$ into intervals on which f, g are injective. How would one have to choose the codomain of f and g so that they are surjective functions? (Of course you need to justify your answer).

- **Problem 4: (recommended)** Use derivatives to show that for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$, one has $|\sin x| \leqslant |x|$. Interpret the inequality in terms of the graphs of $\sin x$ and x. *Hint: Remove the absolute values by considering separately the intervals* $[-\pi/2, 0]$ *and* $[0, \pi/2]$.