PROBLEM SET NO. 6 (DUE ON MONDAY, NOVEMBER 17 AT 4:00 PM)

- **Problem 5:** Find suitable codomains such that the following functions f defined on $\mathbb{R}$ are surjective. Argue that these functions are in fact invertible by explicitly giving f^{-1}.
  (i) $f(x) = x^3 + 1$
  (ii) $f(x) = \begin{cases} x & \text{, if } x \text{ rational,} \\ -x & \text{, if } x \text{ irrational.} \end{cases}$
  (iii) Given n distinct real numbers $a_1, \dots, a_n$, $n \in \mathbb{N}$, let
  $$f(x) = \begin{cases} x & \text{, if } x \neq a_1, \dots, a_n \ , \\ a_{i+1} & \text{, if } x = a_i \ \text{ for } i = 1, \dots, n-1 \ , \\ a_1 & \text{, if } x = a_n \ . \end{cases}$$
  Hint: For parts (ii) and (iii) based on the definition of f make a guess what f^{-1} could be. Then, verify that this guess is indeed the inverse function of f, using the following fact: Given a function $f : A \to B$, where A, B are sets. If there exists a function $g : B \to A$ such that $f \circ g = \mathrm{id}_B$ and $g \circ f = \mathrm{id}_A$, then f is bijective and $g = f^{-1}$. The proof of this fact is short, and just relies on the definition of the inverse; give it a try.
- **Problem 6:** Apostol, Sec. 4.15 problem 8.