PROBLEM SET 5 (DUE ON MONDAY, NOVEMBER 10 AT 4:00 PM)

- **Problem 5:** (recommended) Let I be an open interval on $\mathbb{R}$ and $n \in \mathbb{N}$. A function $f : I \to \mathbb{R}$ is said to be n **times continuously differentiable** (shorthand notation: $f \in \mathcal{C}^n(I)$) if $f^{(n)}$ exists and is continuous on I. Prove that for all $n \in \mathbb{N}$, there exists $f \in \mathcal{C}^n(\mathbb{R})$ such that f is *not* $(n+1)$-times differentiable on $\mathbb{R}$. *Hint: The function*

$$f(x) := \begin{cases} x^2 \sin(1/x) & , \text{ if } x \neq 0, \\ 0 & , \text{ if } x = 0, \end{cases}$$

satisfies the claim for $n = 1$, as we saw in class. Generalize this to arbitrary n.