PROBLEM SET 5 (DUE ON MONDAY, NOVEMBER 10 AT 4:00 PM)

- **Problem 3:**
  (i) Use derivatives to prove that polynomials are uniquely determined by their coefficients, i.e. prove that if two polynomials p, q satisfy $p(x) = q(x)$ for all $x \in \mathbb{R}$, then their degrees are equal and the coefficients corresponding to x^n coincide for all $n \in \mathbb{N}$.
  (ii) An important application of part (i) is the following type of problem, which will be useful when integrating rational functions: Find $A, B \in \mathbb{R}$ such that the following holds for all real $x \neq 1, 3$:
  $$\frac{x-5}{x^2-4x+3} = \frac{A}{x-3} + \frac{B}{x-1} \ .$$
  Explain how you used part (i) in solving for A, B.
  <u>*Hint:*</u> *For part (i), consider the function $f(x) := p(x) - q(x)$, defined for all $x \in \mathbb{R}$. Compute $f^{(n)}$ for all $n \in \mathbb{N}$.*
- **Problem 4:** Assume that there is a function $L\colon (0,\infty) \to \mathbb{R}$ such that $L'(x) = 1/x$ for $x > 0$. Calculate the derivatives of $h(x) = L(\sin(x))$ and $g(x) = (L(x^2))^3$, both defined where $x > 0$.