# PROBLEM SET 5 (DUE ON MONDAY, NOVEMBER 10 AT 4:00 PM)

- **Problem 1:** "Rational powers"
  (a) Given $a \geqslant 0$ and $n \in \mathbb{N}$, define the $n$**th root of** $a$, denoted by $a^{1/n}$, as the *unique non-negative* solution of the equation $x^n - a = 0$. Show that for every $a \geqslant 0$, $a^{1/n}$ exists.
  (b) Let $a > 0$. For $p \in \mathbb{Z}$ and $q \in \mathbb{N}$, define $a^{p/q} := (a^{1/q})^p$; recall, we defined earlier that $a^0 =: 1$ (set 1, problem 5). Prove that for all $r, s \in \mathbb{Q}$, one has
  $$a^r \cdot a^s = a^{r+s} \ .$$
  *Hint: Use the definition in (a) to first establish that $(a^{1/q})^p = (a^p)^{1/q}$. The proof will crucially depend on the "uniqueness" statement in the definition of the nth root. You may use (without proof) the following facts for integer powers which are obvious consequences of their definition:*
  $$(a \cdot b)^n = a^n \cdot b^n \ , \ (a^n)^m = a^{n \cdot m} \ ,$$
  *for every $a > 0$ and $n, m \in \mathbb{Z}$.*
- **Problem 2:** (Recommended) Prove the product rule for derivatives.