PROBLEM SET NO. 4 (DUE ON MONDAY, OCTOBER 27 AT 4:00 PM)

Wednesday 10-22

- **Problem 4:** (recommended)
  (i) Prove that given two real numbers a, b, the following formulas hold true:
  $$\max\{a, b\} = \frac{1}{2}\left[(a + b) + |a - b|\right] \ ,$$
  $$\min\{a, b\} = \frac{1}{2}\left[(a + b) - |a - b|\right] \ .$$
  (ii) Given two continuous functions f, g, use part (i) to show that the functions $\max\{f, g\}$ and $\min\{f, g\}$ are continuous.
- **Problem 5:** Suppose a function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x+y) = f(x)+f(y)$, all $x, y \in \mathbb{R}$, and that f is continuous at 0. Prove that f is continuous on all of $\mathbb{R}$.
- **Problem 6:** Let f be continuous at a point a and $f(a) \neq 0$. Show that $f(x) \neq 0$ locally about a, i.e. show that there exists $\delta > 0$ such that $f(x) \neq 0$ on $(a - \delta, a + \delta)$.
- **Problem 7:** Let $f : [0, 1] \to \mathbb{R}$ be continuous and such that $f(0) = f(1)$. Prove that there is some $c \in [0, 1/2]$ such that $f(c) = f(c + \frac{1}{2})$. (Hint: Look at $g(x) = f(x) - f(x + \frac{1}{2})$.) Conclude that there are, at any time, antipodal points on the earth's equator with the same temperature. (This is problem 6 in Section 5.3 of Introduction to Real Analysis, by Bartle and Sherbert. Don't worry, you don't have to have this book.)