PROBLEM SET NO. 3 (DUE ON MONDAY, OCTOBER 20 AT 4:00 PM)

Wednesday 10-15

- **Problem 4:** (recommended)
  (a) Suppose that $f(x) \leqslant g(x)$ for all x. Prove that $\lim_{x\to a} f(x) \leqslant \lim_{x\to a} g(x)$, provided that these limits exist.
  (b) How could the hypotheses be weakened? *Hint:* The limit is a "local property."
  (c) If $f(x) < g(x)$ for all x, does it necessarily follow that $\lim_{x\to a} f(x) < \lim_{x\to a} g(x)$?
- **Problem 5:** Show that $\lim_{x\to a} f(x) = 0$ if and only if $\lim_{x\to a} |f(x)| = 0$. Show that this isn't true for numbers other than 0.
- **Problem 6:** We define $\lim_{x\to x_0} f(x) = \infty$ to mean that for all $N \in \mathbb{N}$ there is a $\delta > 0$ such that $|f(x)| > N$ whenever x satisfies $0 < |x - x_0| < \delta$. Show that
  (a) $\lim_{x\to 3} 1/(x-3)^2 = \infty$.
  (b) Prove that if for some $c > 0$, $|f(x)| > c$ for all x, and $\lim_{x\to x_0} g(x) = 0$, then
  $$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \infty \ .$$