PROBLEM SET NO. 2 (DUE ON MONDAY, OCTOBER 13 AT 4:00 PM)

Wednesday, October 8

- **Problem 6:** Given a set $A \subseteq \mathbb{R}$, define the *characteristic (or indicator) function of* A by

$$\chi_A : \mathbb{R} \to \{0,1\} \ , \ \chi_A(x) := \begin{cases} 1 & , \text{ if } x \in A \ , \\ 0 & , \text{ if } x \notin A \ . \end{cases}$$

  (a) Find expressions for $\chi_{A\cup B}$, $\chi_{A\cap B}$, and $\chi_{\mathbb{R}\setminus A}$ in terms of χ_A and χ_B.
  (b) Given *any* function $f : \mathbb{R} \to \{0,1\}$, show that there is always a set $A \subseteq \mathbb{R}$ such that $f = \chi_A$.
  (c) Let g be a function defined on $\mathbb{R}$. Prove that $g = g^2$ if and only if $g = \chi_A$ for some set A. (*To be precise, the last equality holds after possibly re-defining the codomain of* g.)
- **Problem 7: (recommended)** A function f with domain and codomain subsets of $\mathbb{R}$ is called *even* if $f(x) = f(-x)$ and *odd* if $f(x) = -f(-x)$, for all x in the domain of f.
  (i) Determine whether $f + g$ is even, odd, or not necessarily either, in the four cases obtained by choosing the functions f and g even or odd.
  (ii) Do the same as in part (i) for $f \cdot g$ and $f \circ g$.
  (iii) Show that every *even* function f can be written as $f(x) = g(|x|)$, for infinitely many functions g.