Problem 8: Suppose the a function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$.

(i) Prove that $f(x_1 + \cdots + x_n) = f(x_1) + \cdots + f(x_n)$.

(ii) Prove that there is $c \in \mathbb{R}$ such that $f(x) = cx$ for all rational $x$ (at this point we are not trying to say anything about $f(x)$ for irrational $x$).

Hint: First try figuring out what $c$ could be by considering $x \in \mathbb{N}$. Then try to prove that the same $c$ works for $x \in \mathbb{Z}$, and finally for $x \in \mathbb{Q}$. 