PROBLEM SET NO. 2 (DUE ON MONDAY, OCTOBER 13 AT 4:00 PM)

Friday, 10-10

- **Problem 8:** Suppose the a function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in \mathbb{R}$.
  (i) Prove that $f(x_1 + \cdots + x_n) = f(x_1) + \cdots + f(x_n)$.
  (ii) Prove that there is $c \in \mathbb{R}$ such that $f(x) = cx$ for all *rational* x (at this point we are not trying to say anything about $f(x)$ for irrational x).
  Hint: First try figuring out what c could be by considering $x \in \mathbb{N}$. Then try to prove that the same c works for $x \in \mathbb{Z}$, and finally for $x \in \mathbb{Q}$.