# PROBLEM SET NO. 1 (DUE ON MONDAY, OCTOBER 06 4:00 PM)

## WEDNESDAY, 10/1

- **Problem 2:**
  (a) Show that if both $a \leq b$ and $-a \leq b$, for some real $a$ and real, non-negative $b$, then $|a| \leq b$.
  (b) Use the *triangle inequality* discussed in class to prove what is known as "*the left hand side of the triangle inequality*": For all real $a, b$,
  $$|(|a| - |b|)| \leq |a + b| \ .$$
  *Hint: Part (a) may come handy for (b).*
- **Problem 3:** Let $\epsilon > 0$ be a fixed positive number. Prove that if $y_0 \neq 0$ and
  $$|y - y_0| < \min\left(\frac{|y_0|}{2}\ ,\ \frac{\epsilon|y_0|^2}{2}\right)\ ,$$
  then $y \neq 0$ and
  $$\left|\frac{1}{y} - \frac{1}{y_0}\right| < \epsilon\ .$$
  Here, given two numbers $a, b$, $\min(a, b)$ denotes the *minimum* of $a$ and $b$. Similarly, one uses $\max(a, b)$ to denote the *maximum* of two numbers $a$ and $b$.

Given an integer $0 \leq n$, one inductively defines $n!$ ("*n factorial*") by $0! = 1$, $n! = n \cdot (n-1)!$ if $n \geq 1$. This is used to define the *binomial coefficient* ("n choose k") for integers $n, k$ with $n \geq 0$,

$$\binom{n}{k} = \begin{cases} \dfrac{n!}{k!(n-k)!} & \text{, if } 0 \leq k \leq n\ , \\ 0 & \text{, if } k > n\ ,\ k < 0\ . \end{cases}$$

$\binom{n}{k}$ appears in combinatorics where it describes the number of ways to choose $k$ items from a collection of $n$, disregarding order (e.g. How many groups of three can one form in a class of 21 students?)

- **Problem 4: (recommended)**
  (a) Prove that for all non-negative integers $n, k$ with $0 \leq k \leq n + 1$ one has
  $$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}\ .$$
  (*You do not need to use induction here.*)

(b) Use induction and the relation from part (a) to convince yourself that $\binom{n}{k}$ is always a non-negative integer.