Problem 2:
(a) Show that if both $a \leq b$ and $-a \leq b$, for some real $a$ and real, non-negative $b$, then $|a| \leq b$.
(b) Use the triangle inequality discussed in class to prove what is known as “the left hand side of the triangle inequality”: For all real $a, b$,
$$ |(|a| - |b|) | \leq |a + b| . $$

Hint: Part (a) may come handy for (b).

Problem 3: Let $\epsilon > 0$ be a fixed positive number. Prove that if $y_0 \neq 0$ and
$$ |y - y_0| < \min \left( \frac{|y_0|}{2}, \frac{\epsilon|y_0|^2}{2} \right) , $$
then $y \neq 0$ and
$$ \left| \frac{1}{y} - \frac{1}{y_0} \right| < \epsilon . $$

Here, given two numbers $a, b$, $\min(a, b)$ denotes the minimum of $a$ and $b$. Similarly, one uses $\max(a, b)$ to denote the maximum of two numbers $a$ and $b$.

Given an integer $0 \leq n$, one inductively defines $n!$ ("$n$ factorial") by $0! = 1$, $n! = n \cdot (n - 1)!$ if $n \geq 1$. This is used to define the binomial coefficient ("$n$ choose $k$") for integers $n, k$ with $n \geq 0$,
$$ \binom{n}{k} = \begin{cases} 
\frac{n!}{k!(n-k)!}, & \text{if } 0 \leq k \leq n , \\
0, & \text{if } k > n , \ k < 0 . 
\end{cases} $$

$\binom{n}{k}$ appears in combinatorics where it describes the number of ways to choose $k$ items from a collection of $n$, disregarding order (e.g. How many groups of three can one form in a class of 21 students?)

Problem 4: (recommended)
(a) Prove that for all non-negative integers $n, k$ with $0 \leq k \leq n + 1$ one has
$$ \binom{n + 1}{k} = \binom{n}{k - 1} + \binom{n}{k} . $$
(You do not need to use induction here.)
(b) Use induction and the relation from part (a) to convince yourself that \(^nC_k\) is always a non-negative integer.