

PROBLEM SET NO. 1 (DUE ON MONDAY, OCTOBER 06 4:00 PM)

1. TUESDAY, 10/01

- **Problem 1*:** Deduce the following statement about real numbers, only using the nine algebraic properties (“field axioms”) and the order axioms introduced in class. Justify each step of your argument by indicating which property you are using. In what follows, a, b, c, d stand for arbitrary real numbers.

(a) $a \cdot d = 0$ if and only if $a = 0$ or $d = 0$.

Remark 1.1. The phrase “*if and only if*” (short hand: “iff”) expresses equivalence of two statements; it thus requires you to verify *two* implications, e.g. for the present example that

(1) $a \cdot d$ implies ($a = 0$ or $d = 0$)

(2) ($a = 0$ or $d = 0$) implies $a \cdot d = 0$

(b) $(-a)b = -(ab)$ and $(-a)(-b) = ab$.

(c) For $a, b \geq 0$ one has $a^2 < b^2$ if and only if $a < b$.

(d) If $a < b + \epsilon$, for *all* numbers $\epsilon > 0$, then one has $a \leq b$. (*Hint: Try a “proof by contradiction” i.e. assume that the statement you need to verify was false (here: suppose $a > b$ in spite of $a < b + \epsilon$ for all $\epsilon > 0$) and show that this assumption invariably leads to a contradiction.*)