Let $L/K$ be an extension of number fields with rings of integers $S/R$. The different of this extension is the set
\[ \text{diff}(S/R) = \{ x \in L | \text{Tr}_{L/K}(xS) \subseteq R \}^{-1}. \]

You may want to consult Ch.III Exercise 33 for a slightly more general construction. By part (c) of this exercise $\text{diff}(S/R)$ is an ideal in $S$ which by problem 3) below is a refinement of the relative discriminant $\text{disc}(S/R)$ (an ideal in $R$).

In the following problems from Marcus’ book you may freely use results of previous exercises with proper quotation.

1) Do Ch. III, exercise 36.

2) Show that $N_{L/K} \text{diff}(S/R) = \text{disc}(S/R)$.

Here the Norm on ideals is defined as $N_{L/K}(\mathfrak{a}) = [S : \mathfrak{a}]_R$ where for two finitely generated, torsion free $R$-modules $N \subseteq M$ of the same rank one defines the "index" $R$-ideal $[M : N]_R$ as the "order ideal" $o(M/N)$ of the finitely generated torsion $R$-module $M/N$. This in turn is defined as
\[ o(M/N) = \prod_p p^{e_p,1 + \cdots + e_p,s_p} \]

where $(M/N)_p \cong R_p/p^{e_p,1} \oplus \cdots \oplus R_p/p^{e_p,s_p} R_p$ is the invariant factor decomposition into cyclic modules given by the structure theorem for modules over the PID $R_p$. You can use w/o proof: If $A_p$ is the matrix expressing an $R_p$-basis of $N_p$ in terms of an $R_p$-basis of $M_p$, then $R_p \cdot \det(A_p) = p^{e_p,1 + \cdots + e_p,s_p} R_p$.

3) Do Ch. IV exercise 27

4) Recall that a prime $p$ is said to split completely in an extension $L/K$ if $\mathcal{O}_L \cdot p$ is a product of $[L : K]$ distinct primes. Show that a prime $p$ splits completely in $L/K$ if and only if it splits completely in $N/K$ where $N/K$ is the normal closure of $L/K$. 