1. (i) Find a maximum matching (by hand) in the graph $G_0$ that will be provided as a Handout on the web site soon. You can start with any matching and use the labeling procedure to be sure there is no matching of larger size.

(ii) Find a minimum vertex-cover in $G_0$. Do this with the labeling procedure.

2. Complete the proof of Theorem 5 of Notes #11. Suggestion: Start by explaining why, if the weighting $w$ (with $\delta w$ integer-valued) is not itself integer-valued, then there exists a polygon $P$ so that $w(e)$ is not an integer for all $e \in E(P)$.

3. Describe an algorithm for finding a polygon in an (undirected) graph $G$ (with thousands of vertices), if there are any. You may be brief, but try to be precise and complete. (I expect all kinds of answers to this.)

4. [NO COLLABORATION.] (i) Prove that if a balanced digraph $D$ is connected, then it is strongly connected.

(ii) Given a digraph $D$ and two distinct vertices $s$ and $t$, prove that either (a) there exists a directed path from $s$ to $t$, or (b) there exists a cut $(X, Y)$ separating $s$ and $t$ such that there are no edges of $D$ from $X$ to $Y$.

5. Let $D$ be a digraph with two distinct vertices $s$ and $t$, and where each (directed) edge $e$ has a capacity $c(e)$. Suppose $(X_1, Y_1)$ and $(X_2, Y_2)$ are both cuts separating $s$ and $t$ in $D$ of minimum capacity in $D$ (“mincuts”). Prove that $(X_1 \cap X_2, Y_1 \cup Y_2)$ is also a mincut. Give a simple example where $X_1 \cap X_2$ is not equal to $X_1$ or $X_2$. 