Arctic Circles, Domino Tilings and Square Young Tableaux

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Abstract. It is well known that domino tilings and certain other random combinatorial or statistical-physics models on a two-dimensional lattice exhibit a spatial phase transition between an “arctic” or “frozen” region where the behavior of the random object is asymptotically deterministic and a “temperate” region where the entropy of the system is concentrated. One famous example of such a phenomenon is the Arctic Circle Theorem due to Jockusch, Propp and Shor, which shows that for uniformly random domino tilings of a particular region known as the Aztec Diamond, the curve that forms the interface between the frozen and temperate regions converges to a circle. Cohn, Elkies and Propp later derived a more detailed result about the limiting height function of the typical domino tiling of the Aztec diamond. In this talk, I will present a new proof of the Cohn–Elkies–Propp limit shape result for the height function based on a connection to alternating-sign matrices and a variational analysis. The proof highlights a surprising connection of this result to another arctic-circle type phenomenon observed in a different problem involving uniformly random square Young tableaux.