

Caltech Mathematical Physics Seminar
Spring 2008–09

Asymptotics of OPRL for a Weight with a Jump: (Lack of) Universality and Clock Behavior

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Abstract. We consider the orthogonal polynomials on $[-1, 1]$ with respect to a generalized Jacobi weight modified by a step-like function. We obtain strong uniform asymptotics of the monic orthogonal polynomials in the whole plane, as well as first terms of the asymptotic expansion of the main parameters (leading coefficients of the orthonormal polynomials and the recurrence coefficients). In particular, we prove a conjecture of A. Magnus regarding the asymptotics of the recurrence coefficients.

The main focus is on the local analysis at the origin. We study the asymptotics of the Christoffel–Darboux kernel in a neighborhood of the jump and show that the zeros of the orthogonal polynomials no longer exhibit the clock behavior. For the asymptotic analysis we use the steepest descendent method of Deift and Zhou applied to the non-commutative Riemann–Hilbert problems characterizing the orthogonal polynomials. The local analysis at the jump is carried out in terms of the confluent hypergeometric functions. Incidentally, we establish some properties of these functions that may have an independent interest.

This is a joint work with A. Foulquie and V. Sousa (Univ. of Aveiro, Portugal).