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Jacobi Matrices and Their Generalizations in Padé Approximation

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Abstract. It is well known that Jacobi matrices are intimately related to moment problems and can be used to prove convergence results for Padé approximants. We consider two generalizations of the scheme.

First, we expand rational perturbations of Nevanlinna–Herglotz functions into continued fractions of a special type called P-fractions. In fact, the P-fractions are a natural generalization of Jacobi fractions and it turns out that the n-th convergent to a P-fraction coincides with the n-th diagonal Padé approximant to the function in question. Next, we introduce generalized Jacobi matrices (a block tridiagonal matrix) associated with the P-fractions. Then, convergence of Padé approximants appears as the resolvent convergence of finite matrix approximations to the original generalized Jacobi matrix. It allows us to prove the locally uniform convergence of diagonal Padé approximants for the functions under consideration.

Second, we turn our attention to Nevanlinna–Pick problems (actually, a moment problem is a limiting case of Nevanlinna–Pick problems) and we present a modification of the well-known step-by-step process for solving Nevanlinna–Pick problems in the class of Nevanlinna–Herglotz functions. The modified process gives rise to a linear pencil $H - xJ$, where H and J are Hermitian tridiagonal matrices. Moreover, J is a positive operator and it should also be mentioned that when a Nevanlinna–Pick problem reduces to a moment problem, the corresponding operator J becomes the identity operator. As in the classical scheme, we relate the spectral theory of the pencil $H - xJ$ and the corresponding Nevanlinna–Pick problem. In particular, locally uniform convergence of multipoint Padé approximants is shown.