Improving the Consistency Strength of Reflection at $\aleph_{\omega+1}$

Abstract: Reflection at $\aleph_{\omega+1}$, the statement that every stationary subset of $\aleph_{\omega+1}$ has a reflection point below $\aleph_{\omega+1}$, was shown to be consistent given the consistency of infinitely many supercompact cardinals by Magidor in 1982. In this talk we will consider improving the known upper bound for the consistency to the existence of a cardinal κ which is κ^+ supercompact.

If κ is κ^+ supercompact then every stationary subset of $\kappa^+ \setminus \mathrm{cof}(\kappa)$ has a reflection point of cofinality less than κ. Using a Modified Prikry forcing similar to the one used by Woodin to get the failure of SCH at $\aleph_\omega$ we can turn κ^+ in to $\aleph_{\omega+1}$ giving:

Theorem. *(GCH) Assume κ is κ^+ supercompact and $\mathbb{P}$ is a modified Prikry forcing at κ. Then in the generic extension, $\kappa = \aleph_\omega$, $(\kappa^+)^V = \aleph_{\omega+1}$ and every stationary $S \subseteq \aleph_{\omega+1}$ not concentrating on points of V-cofinality κ has a reflection point.*

This results will be presented as well as possible strengthenings of this result.