somatic mesoderm of the chorion. The two membranes together are known as the chorioallantoic membrane (illustration d).

Function. The amniotic cavity within which the embryo is now enclosed becomes filled with an aqueous fluid which gives osmotic and physical protection to the embryo during the remainder of its fetal existence. Smooth muscle fibers in the amnion spontaneously contract and gently rock the embryo before it develops the capacity for spontaneous movement.

Nutrients and wastes. Numerous blood vessels originate in the splanchnic mesoderm associated with the endoderm of both the yolk sac and the allantois and anastomose to a form an extraembryonic vascular network. Soluble nutrients from the volk (or from the maternal blood in the case of placental mammals) are absorbed into these blood vessels and are carried to the embryo, where they are utilized to provide energy for the biological processes involved in growth and development. Water, carbon dioxide, and nitrogenous wastes are by-products of these biological processes. Carbon dioxide and nitrogenous wastes are toxic to the embryo and must somehow be separated from it if it is to continue normal development. It is the allantois, in close relationship with the chorion, which functions in the effective removal of these wastes.

Gaseous exchange. Carbon dioxide diffuses into the bloodstream, and the chorioallantoic blood vessels bring it close to a source of oxygen for which it can be exchanged. This exchange takes place through the shell or, in mammals, through the maternal blood vessels of the placenta.

Nitrogenous wastes. Nitrogenous wastes are carried away from the embryo in the allantoic artery. In birds and reptiles these waste products consist primarily of uric acid, which is relatively insoluble in water. Before the concentration of uric acid reaches a toxic level, it precipitates out of solution and is stored in the cavity of the allantois, where it causes no harm to the embryo. The primary nitrogenous waste in mammals is urea, which is both soluble and toxic but is continuously removed by way of the allantoic (umbilical) arteries to the placenta where it diffuses into the maternal blood vessels and is eventually excreted by the kidneys of the mother.

**Fate.** As the stored nutrients of the yolk are depleted in the course of development, the yolk sac gradually decreases in size and is eventually incorporated into the mid-gut of the embryo. The yolk sac in the nonyolky eggs of placental mammals is vestigial. It has evolutionary but essentially no functional significance.

At the time of birth or hatching, the embryo becomes completely separated from the amnion and chorion and from the major portion of the allantois. The proximal portion of the latter remains within the embryo, however, as the urinary bladder. See Allantois; Amnion; CHORION; YOLK SAC.

## [ANNA RUTH BRUMMETT]

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## Fever

An elevation in the central body temperature of warm-blooded animals caused by abnormal functioning of the thermoregulatory mechanisms. Fever accompanies a wide variety of disease states, both infectious and noninfectious, and in the great majority of instances is due to an abnormality in the regulation of body temperature by the central nervous system. See THERMOREGULATION.

Endogenous pyrogens, Experimental studies on the cause of fever suggest that leukocytes (circulating granulocytes and monocytes) of the host, as well as the fixed macrophages of the reticuloendothelial system in the liver and spleen can be activated by various stimuli to produce a feverinducing substance-endogenous pyrogen (EP). Activators of microbial origin include the lipopolysaccharide endotoxins of gram-negative bacteria. cell walls and soluble antigens of other bacteria. viruses, and fungi. Endogenous pyrogen has been characterized as a protein of relatively low molecular weight (13,000) with an essential lipid moiety. Pyrogens from different tissues and from rabbits and humans appear to have similar characteristics. Circulating EP, liberated by activated cells, is believed to cross the blood-brain barrier and act upon specialized neurons, presumably located in the anterior hypothalamus, that initiate heat conservation and increase heat production.

**Mechanisms.** The mechanisms responsible for elevating body temperature include: reduction in heat loss by constriction of peripheral vessels whose tone is under control of the sympathetic nervous system; inhibition of panting and sweating, the latter by way of the cholinergic nerves; and increased heat production by means of shivering in voluntary muscles innervated by somatic motor nerves.

**Pharmacologic pyrogens.** Various naturally occurring pharmacologic agents, including serotonin (5-hydroxytryptamine) and catecholamines, are normally present in high concentration in areas of the brain that control body temperature and can cause fever when injected under certain experimental circumstances. The role of these agents in most experimentally produced or clinical fevers is obscure. Certain naturally occurring steroids are also pyrogenic and are known to activate human leukocytes in the test tube.

**Effects.** There is no clear evidence that elevated body temperature evoked by most infections is directly injurious to microbial invaders. As fever regularly accompanies inflammation, however, increased body temperature may well accelerate certain biochemical reactions of use to the host in combating infection. *See* HOMEOSTASIS: INFLAM-MATION. [ELISHA ATKINS]

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## Feynman integral

A technique, also called the sum over histories, which is basic to understanding and analyzing the dynamics of quantum systems. It is named after fundamental work of Richard Feynman. The crucial formula gives the quantum probability density for transition from a point  $q_o$  to a point  $q_1$  in time t as expression (1), where S(path) is the classical me-

$$\exp\left[iS(\text{path})/\hbar\right]d(\text{path}) \tag{1}$$

chanical action of a trial path, and  $\hbar$  is the rationalized Planck's constant. The integral is a formal one over the infinite-dimensional space of all paths which go from  $q_0$  to  $q_1$  in time t. Feynman defines it by a limiting procedure using approximation by piecewise linear paths. See ACTION.

Feynman integral ideas are especially important in quantum field theory, where they not only are a useful device in analyzing perturbation series but are also one of the few nonperturbative tools available.

An especially attractive element of the Feynman integral formulation of quantum dynamics is the classical limit,  $\hbar \rightarrow 0$ . Formal application of the method of stationary phase to expression (1) says that the significant paths for small  $\hbar$  will be the paths of stationary action. One thereby recovers classical mechanics in the hamiltonian stationary action formulation. See LEAST-ACTION PRINCIPLE.

**Feynman-Kac formula.** Mark Kac realized that if one replaces time by a formal purely imaginary time (that is, replaces the Schrödinger equation by a heat equation), then one can make precise mathematical sense out of Feynman's integral in terms of the stochastic process introduced by Norbert Wiener in his theory of Brownian motion. The resulting formula for the semigroup  $e^{-tH}$ , with H= $-\frac{1}{2}\Delta + V$  the quantum hamiltonian ( $\Delta$  is the laplacian operator and V is the potential energy), is given by Eq. (2). Here  $E_x$  is the expectation with

$$(e^{-tH}f)(x) = E_x \left( \exp\left\{ -\int_0^t V[b(s)]ds \right\} f[b(t)] \right)$$
(2)

respect to Wiener's process, b(s), starting at the point x, and f is an arbitrary bounded measurable function to which  $e^{-tH}$  is applied. Equation (2) is called the Feynman-Kac formula. It makes a wide variety of probabilistic ideas applicable to a rigorous mathematical analysis of quantum theory. See BROWNIAN MOVEMENT; STOCHASTIC PROCESS.

**Euclidean quantum field theories.** The passage to imaginary time initiated by Kac is especially useful in relativistic quantum field theory. Since the Lorentz covariance of relativity is replaced by euclidean group covariance under this replacement, the new theory is often called euclidean field theory. The corresponding Feynman integral is often called a euclidean path integral and has become the standard formulation of Feynman integrals in most modern discussions of quantum field theory. The analogs of stationary action solutions are called instantons. See INSTANTONS.

**Statistical mechanics analogy.** The appearance of the exponential in Eq. (2) suggests an analogy between quantum dynamics and classical statistical mechanics. The analog of the total spacetime dimension in a euclidean quantum field theory is then the dimension of space in statistical mechanics. Just as the phenomenon of phase transitions occurs in statistical mechanics, the analogous phenomena of spontaneously broken symmetries and dynamical instability occur in quantum field theory. These ideas are a basic element of all modern theories of elementary particle physics. They are usually analyzed within a Feynman integral framework. *See* ELEMENTARY PARTICLE; NON-RELATIVISTIC QUANTUM THEORY; PHASE TRANSI-TIONS; QUANTUM FIELD THEORY; STATISTICAL MECHANICS; SYMMETRY LAWS (PHYSICS).

[BARRY SIMON]

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## Fiber bundle

A type of structure arising frequently in various branches of mathematics, especially differential geometry, Lie groups, and algebraic geometry. A fiber bundle is a principal tool for the applications of algebraic topology to these subjects.

A fiber bundle is a topological space B which is decomposed in a smooth fashion into a family of closed disjoint subsets, called fibers, each of which is homeomorphic to a fixed space Y. The base space X of the bundle is the space whose points are the fibers. The projection  $p: B \to X$  is the function assigning to each point the fiber which contains it, and X is topologized so that p is continuous. The requirement of smoothness reads as follows: For each  $x \in X$ , there is a neighborhood V of x, and a homeomorphism  $\varphi: V \times Y \to p^{-1}(V)$ satisfying  $p\varphi(x', y) = x'$  for all  $x' \in V$  and  $y \in Y$ . See TOPOLOGY.

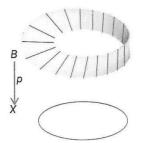
The product space  $X \times Y$  and its projection into X satisfy these conditions with each V=X. The Möbius band (see illustration) is one of the simplest examples of a bundle which is not a product. The fibers are the line segments perpendicular to its center line, and its base space is a circle. The corresponding product space is a cylindrical surface. Because of such examples, fiber bundles are sometimes called twisted products.

Fiber bundles arise naturally in several different ways. If B is a Lie group and Y is a closed subgroup of B, then the family of left cosets of Y in B is a fibration of B, and the base space X is a manifold on which B is a transitive transformation group. For example, let B be the group of all rotations of a two-dimensional sphere, and let Y be the subgroup of rotations about a fixed axis. Then B is a threedimensional manifold (projective three-space), Y is a circle, and X is the sphere.

Let X be a differentiable manifold. The vectors tangent to X at a point  $x \in X$  form a linear space  $Y_r$ . Their union for all  $x \in X$  is a differentiable manifold B called the tangent bundle of X. The projection p assigns to each vector its initial point. Similarly, there is a tensor bundle over X for each algebraic type of tensor (for example, skew-symmetric tensors of two covariant indices with nonzero determinant).

A cross section of a bundle B is a mapping  $f: X \to B$  such that pf(x) = x for each  $x \in X$ . In case  $B = X \times Y$ , f is the graph of a mapping  $X \to Y$ . In case X is a differentiable manifold and B is a tensor bundle over X, then f is a tensor field.

FIBER BUNDLE



Projection of topological space *B* (a Möbius band) into its base space *X* (a circle).